Non-Darcy Flow and Wellbore Storage Effects in Pressure Build-Up and Drawdown of Gas Wells

H. J. Ramey, Jr.  
MEMBER AIME  
COLLEGE STATION, TEX.

ABSTRACT

The wellbore acts as a storage tank during drawdown and build-up testing and causes the sand-face flow rate to approach the constant surface flow rate as a function of time. This effect is compounded if non-Darcy flow (turbulent flow) exists near a gas wellbore. Non-Darcy flow can be interpreted as a flow-rate dependent skin effect. A method for determining the non-Darcy flow constant using this concept and the usual skin effect equation is described. Field tests of this method have identified several cases where non-Darcy flow was severe enough that gas wells in a fractured region appeared to be moderately damaged.

The combination of wellbore storage and non-Darcy flow can result in erroneous estimates of formation flow capacity for short-time gas well tests. Fortunately, the presence of the wellbore storage effect permits a new analysis which can provide a reasonable estimate of formation flow capacity and the non-Darcy flow constant from a single short-time test.

The basis of the Gladfelter, Tracy and Wilsey correction for wellbore storage in pressure build-up was investigated. Results led to extension of the method to drawdown testing. If non-Darcy flow is not important, the method can be used to correct short-time gas well drawdown or build-up data. A method for estimation of the duration of wellbore storage effects was developed.

INTRODUCTION

In 1953, van Everdingen and Hurst generalized results published in their previous paper concerning wellbore storage effects to include a “skin effect”, or a region of altered permeability adjacent to the wellbore. Later, Gladfelter, Tracy and Wilsey presented a method for correcting observed oilwell pressure build-up data for wellbore storage in the presence of a skin effect. The method depended upon measuring the change in the fluid storage in the wellbore by measuring the rise in liquid level.

To the author’s knowledge, application of the Gladfelter, Tracy and Wilsey storage correction to gas-well build-up has not been discussed in the literature. It is, however, a rather obvious application. Gas storage in the wellbore is a compressibility effect and can be estimated easily from the measured wellbore pressure as a function of time.

Several approaches to the wellbore storage problem have been suggested. As summarized by Matthews, it is possible to minimize annulus storage volume by using a packer, and to obtain a near sand-face shut-in by use of down-hole tubing plug devices. Matthews and Perrine have suggested criteria for determining the time when storage effects become negligible.

In 1962, Swift and Kiel presented a method for determination of the effect of non-Darcy flow (often called turbulent flow) upon gas-well behavior. This paper provided a theoretical basis for peculiar gas-well behavior described previously by Smith.

Recently, Carter, Miller and Riley observed disagreement among flow capacity data determined from gas-well drawdown tests conducted at different flow rates for short periods of time (less than six hours flowing time). In the original preprint of their paper, Carter et al. proposed that the discrepancy in flow capacity was possibly a result of wellbore storage effects. Results of an analytical study of unloading of the wellbore and non-Darcy flow were recorded by Carter. In the final text of their paper, Carter et al. stated that they no longer believed wellbore storage was the reason for discrepancy in their data estimates.

In view of the preceding, this study was performed to establish the importance of non-Darcy flow and wellbore storage for gas-well testing. In the course of the study, a reinspection of the previous work by van Everdingen and Hurst was made, and the basis for the Gladfelter, Tracy and Wilsey wellbore storage correction was investigated and extended to flow testing.

WELLBORE STORAGE THEORY

As has been shown by Aronofsky and Jenkins, Matthews, and others, flow of gas can often be approximated by an equivalent liquid flow system. The following development will use liquid flow nomenclature to simplify the presentation. Application to gas-well cases will be illustrated later. First, we will use the van Everdingen-Hurst treatment of wellbore storage in transient flow to establish (1) the duration of wellbore storage effects, and (2) a method to correct flow data for wellbore storage.

DURATION OF WELLBORE STORAGE EFFECTS

When an oil well is opened to flow, the bottom-hole pressure drops and causes a resulting drop in the liquid level in the annulus. If $V$, the annular volume in cu ft/ft of depth, and $\rho$ represents the average density of the fluid in the wellbore, the volume of fluid at reservoir conditions produced from the annulus per unit bottom-hole pressure drop is approximately:

$$C_{\text{res}} \frac{\text{bbl}}{\text{psi}} = \frac{(V \times \text{cu ft/ft}) (144 \text{ sq in.}/\text{sq ft})}{(5.615 \text{ cu ft/bbl}) (\rho \text{ lb/cu ft})}$$
All factors in Eq. 1 are constant except the density term, which is an average value between the initial pressure and the final drawdown pressure. Thus, $C_n$, the reservoir barrels of fluid unloaded from the annulus per unit bottom-hole pressure drop, is essentially a constant as described by van Everdingen and Hurst. We can write a similar expression for the storage of gas in a wellbore. If \( V_n \) is the total wellbore volume (both annulus and tubing), and $c$ is the average compressibility of gas between initial pressure and final drawdown pressure:

$$C_n \text{ res cu ft/psi}} = \frac{(V_n \text{ cu ft})(c \text{ vol/vol-psi})}{141.4 \text{ qpB}} \quad \cdots \cdots \cdots \quad (5)$$

It is apparent then, that the volume of liquid or gas unloaded from the wellbore per unit pressure drop is essentially a constant. If we drop the subscript on $C$ for the time being, the rate of unloading of the wellbore can be expressed as:

$$q_t = \frac{C}{B} \frac{d(p_i - p_{ew})}{dt} \quad \cdots \cdots \cdots \quad (3)$$

If the well is produced at a constant rate $q$ at the surface, then the sand-face production rate becomes:

$$q_{st} = q - q_s \quad \cdots \cdots \cdots \quad (4)$$

From Eq. 3, the wellbore unloading rate will depend upon time. Basically then, we wish to calculate the pressure drawdown if a well is produced at constant rate at the surface; but unloading of the wellbore causes the sand-face production rate to increase from zero to the constant surface rate, as indicated by Eqs. 3 and 4. This leads to an integro-differential equation presented by van Everdingen and Hurst as their Eq. VIII-3. Solutions have been presented by van Everdingen and Hurst as dimensionless pressure drawdown, $p_h(t_n)$, and are reproduced in Fig. 1. The dimensionless wellbore storage constant $\bar{C}$ is related to the oil and gas wellbore storage constants given in Eqs. 1 and 2 as follows.

For oil:

$$\bar{C} = \frac{(5.615 \text{ cu ft/bbl})(C \text{ res bbl/psi})}{2\pi \phi h c r_s^3} \quad \cdots \cdots \cdots \quad (5)$$

For gas:

$$\bar{C} = \frac{(C \text{ res cu ft/psi})}{2\pi \phi h c r_s^3} \quad \cdots \cdots \cdots \quad (6)$$

![Fig. 1 - Dimensionless Pressure Drop vs Dimensionless Time Including Wellbore Storage Effect (After van Everdingen and Hurst, Refs. 1, 2 and 3).](image)

Engineering units are used throughout. Note that the compressibility $c$ presented in Eqs. 5 and 6 should be interpreted as the effective compressibility of the total reservoir system, not the compressibility of the fluid in the wellbore. This compressibility term arises in converting to dimensionless time in the integro-differential equation.

The dimensionless pressure drawdown, $p_h(t_n)$ is defined as follows.

For oil:

$$p_h(t_n) = \frac{kh(p_i - p_{ew})}{141.4 \text{ qpB}} \quad \cdots \cdots \cdots \quad (7)$$

where $q$ is in units of STB/D, and for gas:

$$p_h(t_n) = \frac{kh(p_i - p_{ew})}{25,200 \text{ qpB}} = \frac{T \cdot kh(p_i - p_{ew})}{50,400 \text{ pqrzT}} \quad \cdots \cdots \cdots \quad (8)$$

where $q$ is in units of Mscf/D.

The dimensionless time is defined by

$$t_n = \frac{0.00634 \cdot q t}{k} \quad \cdots \cdots \cdots \quad (9)$$

From Eqs. 3 through 9, it follows that:

$$q_{st} = q \left[1 - \bar{C} \frac{dp_h(t_n)}{dt_n}\right] \quad \cdots \cdots \cdots \quad (10)$$

where $q$, $q_s$, and $q$ may be in any consistent set of units.

Van Everdingen and Hurst later presented similar analyses of the wellbore storage effect including the presence of a skin effect $S$. Regardless of the magnitude of the skin effect and when time is large enough, the pressure drawdown eventually approaches that of constant rate production. Note on Fig. 1 that the dimensionless pressure drawdown, $p_h(t_n)$, values eventually coincide with the $p_h(t_n)$ values for constant rate production at large enough values of dimensionless time. If the time of coincidence is plotted against the dimensionless wellbore storage constant $\bar{C}$, Fig. 2 results. Fig. 2 may be used to

![Fig. 2 - Duration of Wellbore Storage Effects vs Storage Constant (Skin Effect = 0).](image)
approximate the duration of wellbore storage effects on pressure drawdown or build-up. To do this, $\bar{C}$ can be calculated from the appropriate Eqs. 1, 2, 5 and 6, the dimensionless time $t$, found from Fig. 2, and real time then computed from Eq. 9.

Because the line on Fig. 2 can be represented analytically, it is possible to write the following equation for the estimated time when wellbore storage effects become negligible.

$$t = 9.570 \frac{\phi \mu r_0 \bar{C}}{k}$$
$$= 8.560 \frac{\mu C_i}{kh} = 1.525 \frac{\mu C_i}{kh} \quad \ldots \ldots (11)$$

Eqs. 1, 2 and 11 may be used to estimate the time when wellbore storage effects should be negligible. A sample calculation is provided in the Appendix.

Gas-well drawdown presents an interesting special case. In many cases, the effective compressibility of the total reservoir system $c$ can be approximated by $(\bar{S}v_r)$. Combining Eqs. 2 and 6 and approximating the wellbore volume as

$$V_0 = \pi r_0^2 L_i \quad \ldots \ldots \ldots (12)$$

where $L_i$ is the length of the well, leads to:

$$\bar{C} = \frac{L_i}{2 \phi k \bar{S}_r} \quad \ldots \ldots \ldots (13)$$

and Eq. 11 becomes:

$$t = 4.785 \frac{\mu C_i r_0 L_i}{kh} \quad \ldots \ldots \ldots (14)$$

Thus, for gas wells where the casing-tubing annulus is not packed off, the duration of wellbore storage effects can be estimated easily.

As an example, Eq. 14 can be used to produce a table of the duration of wellbore storage effects as a function of the depth of the well and the pressure level. Table 1 presents this type of information for a well completed with 6%-in. liner. Other conditions selected are shown in the table heading.

Table 1 can be used for rapid estimates of the duration of wellbore storage effects for other conditions than those used. Time is inversely proportional to the formation flow capacity $kh$. Thus the estimated duration of wellbore storage effects will increase if $kh$ is less than the 100 md-ft used in the example, and decrease if $kh$ is greater than 100 md-ft.

Inspection of Table 1 reveals interesting trends. Clearly, wellbore storage effects should be most important for low-pressure, moderately deep formations. It is also clear that wellbore storage can cause changing sand-face flow rates during an entire flow test for a well 5,000 ft deep and a pressure level of 1,000 to 2,000 psi. Storage effects may persist for times on the order of a few hours for the high-pressure, deep formations.

In general, wellbore storage effects are likely to be of importance for tests of short duration—less than one day. It is doubtful that wellbore storage will be an important factor in long-time build-up or drawdown testing.

The preceding estimate of the duration of wellbore storage effects is based on a skin effect of zero, or an undamaged or unstimulated well. Evaluation of the effect of a skin on the duration of wellbore storage effects indicates in general that a negative skin effect leads to reduced times, while a positive skin effect increases the duration of wellbore storage effects. Although it is possible to produce other lines on Fig. 2 with the magnitude of the skin effect as a parameter, it does not appear useful to do so. Thus, estimates of the duration of wellbore storage effects should be considered "order of magnitude" estimates if the skin effect is considerably different from zero.

CORRECTION OF FLOW TEST DATA FOR WELLBORE STORAGE

Van Everdingen" and Hurst* introduced another approach to wellbore storage that can lead to further useful conclusions. They observed that in many cases the sand-face (formation) flow rate can be approximated by a formula of the type:

$$q_{s_i} = q(1 - e^{-bt}) \quad \ldots \ldots \ldots (15)$$

where $\beta$ is a positive, dimensionless constant. Eq. 15 indicates that the sand-face flow rate starts from zero and increases exponentially to the final constant rate $q$. Van Everdingen and Hurst employed the superposition theorem to derive the following expression for the pressure transients caused by a changing sand-face flow rate:

$$p_n(t_o) = p_o(t_o) + S - \frac{t_o}{2} e^{\beta t_o}$$

In Eq. 16, $y$ is Euler's constant, 0.57722, and the exponential integral of a positive argument is $\text{Ei}(x)$:

$$\text{Ei}(\beta t_o) = \int_0^{\beta t_o} \frac{e^u}{u} du \quad \ldots \ldots \ldots (17)$$

Van Everdingen and Hurst evaluated the $p_n(t_o)$, which are reproduced on Figs. 1 and 3 as functions of $\beta$. It is apparent that the drawdown curves of $p_n(t_o)$ have similar

\begin{table}[h]
\centering
\small
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Depth of \quad & 0.500 & 1.000 & 2.000 & 5.000 & 10,000 \\
well (ft) & 1,000 & 1,000 & 5,000 & 10,000 & 10,000 \\
& & & & & \\
6,000 & 14 & 14 & 14 & 14 & 14 \\
5,000 & 17 & 17 & 17 & 17 & 17 \\
4,000 & 20 & 20 & 20 & 20 & 20 \\
3,000 & 23 & 23 & 23 & 23 & 23 \\
2,000 & 26 & 26 & 26 & 26 & 26 \\
1,000 & 29 & 29 & 29 & 29 & 29 \\
500 & 32 & 32 & 32 & 32 & 32 \\
0 & 35 & 35 & 35 & 35 & 35 \\
\hline
\end{tabular}
\caption{Estimated Duration of Wellbore Storage Effects for a Gas-Well Drawdown Test.}
\end{table}
lar shapes to curves of \( \frac{q}{q_{in}} \). Although \( \beta \) is an empirical constant and cannot be estimated as readily as the parameter \( C \), the solutions for this case can be used to achieve useful conclusions regarding the Gladfelter, Tracy and Wilsey correction for wellbore storage during pressure build-up and drawdown.

Gladfelter, Tracy and Wilsey\(^1\) performed an analysis of the pressure build-up for a well which had been produced at a constant rate \( q \) for a time \( t \) and then shut in such that the flow rate decreased steadily from rate \( q \) through rates \( q_{in} \) at shut-in times \( t \) until the rate \( q_{in} \) was zero. These authors did not consider the effect of unloading the annulus during the initial drawdown stage. The result of their analysis was a series which the authors found could be simplified to:

\[
(p_{oi} - p_{wi})(\frac{q}{q_{in}}) = \frac{m}{2.303} \left[ \ln \frac{0.00634 k t_{w}}{\phi \mu \tau_{w}} + 0.80907 + 2S \right], \tag{18}
\]

where \( m = \frac{162.6 \phi \mu B}{k \mu h} \). \tag{19}

Gladfelter, Tracy and Wilsey stated the approximate Eq. 18 could be used to correct pressure build-up data for wellbore storage and produce values of the build-up slope \( m \) that were within 1 per cent of the long-time value achieved when wellbore storage effects were no longer important.

An alternate expression for the pressure difference in Eq. 18 can be written using the van Everdingen\(^2\) and Hurst\(^3\) \( p_{o}(t) \) function given by Eq. 16. A comparison of the result with Eq. 18 leads to the conclusion that the Gladfelter, Tracy and Wilsey approximation for pressure build-up, Eq. 18, should be good as long as \( \beta \) is sufficiently small. The comparison is presented in the Appendix.

Of more interest, however, the result leads to the following approximate relationship for pressure drawdown in the presence of wellbore storage effects:

\[
(p_{oi} - p_{wi})(\frac{q}{q_{in}}) = \frac{m}{2.303} \left[ \ln \frac{0.00634 k t_{w}}{\phi \mu \tau_{w}} + 0.80907 + 2S \right] \tag{20}
\]

This equation shows that multiplication of the pressure drawdowns by the appropriate ratio of the constant surface production rate to the sand-face rate should yield a linear function of the flowing times. The slope of the straight line \( m \) can then be used with Eq. 19 to obtain flow capacity in the normal manner. Eq. 20 can be used for gas flow as suggested by Matthews\(^5\) or an alternate expression may be used as follows:

\[
(p_{oi} - p_{wi})(\frac{q}{q_{in}}) = \frac{m'}{2.303} \left[ \ln \frac{0.00634 k t_{w}}{\phi \mu \tau_{w}} + 0.80907 + 2S \right] \tag{21}
\]

where

\[
m' = \frac{46.3 \phi \mu T_{s} p_{oi}}{k h T_{w}}, \tag{22}
\]

The dimensionless skin effect \( S \) used in this paper is that originally described by van Everdingen\(^2\) and Hurst\(^3\) and is numerically one-half the \( S \) used by Gladfelter, Tracy and Wilsey\(^1\).

or

\[
m' = 1.637 \frac{q_{o} T_{s}}{k h} \tag{22-A}
\]

where \( p_{oi} \) is 14.7 psia and \( T_{w} \) is 5200 R, and \( m' \) is the straight line slope of a plot of pressure squared vs log of flowing time. The \( \tau_{w} \) is evaluated at the average pressure

\[
p = \sqrt{\frac{1}{2}(p_{oi}^2 + p_{wi}^2)}, \tag{23}
\]

as presented by Carter.\(^6\)

To use Eqs. 20 or 21, it is necessary to find the sand-face flow rate, \( q_{in} \), as a function of flowing time. For oil flow, the procedure outlined by Gladfelter, Tracy and Wilsey\(^1\) may be used. For gas flow, the rate of decrease in storage of gas in the wellbore can be estimated as:

\[
q_{t} = \frac{d}{dt} \left( \frac{V_{w}}{B_{p}} \right) \frac{Mscf}{D}, \tag{24}
\]

where

\[
B_{p} = \frac{p_{w} Z_{w} T_{w}}{T_{p} p_{wi}} \mathrm{res} \ vol \tag{25}
\]

\[
T_{w} \mathrm{represents the average wellbore temperature in °R. This may be estimated as the arithmetic average of surface and formation temperature.}
\]

Substitution of Eqs. 24 and 25 in Eq. 4 leads to:

\[
q_{in} = q - \frac{V_{w} T_{w}}{1,000 T_{p} p_{wi}} \left( \frac{dp_{w}}{dt} \right) \approx q - \frac{V_{w} T_{w}}{1,000 T_{p} p_{wi}} \left( \frac{\Delta p_{w}}{\Delta t} \right). \tag{26}
\]

The gas-law deviation factor \( Z_{w} \) is evaluated at the average temperature \( T_{w} \) and the pressure \( p_{wi} \). This approximation does not consider the change in pressure with depth, although it could be done easily.\(^*\)\(^*\) Eq. 26 can be used with information normally available in any gas-well flow test to estimate the sand-face flow rate and thus permit use of either Eqs. 20 or 21 to make correction for wellbore storage effects.

In view of the fact that the wellbore storage correction given by Eqs. 20 and 21 was based on an empirical representation of the sand-face flow rate, a legitimate question can be raised as to the utility of the correction. One test of the method can be made by applying the correction to the data published by Carter.\(^6\) Carter computed gas-well drawdown data including wellbore storage effects and non-Darcy flow effects. In addition to the computed drawdown data, Carter also presented the sand-face flow rates as functions of flowing time. Table 2 presents a comparison of the capacity \( (k h) \) values obtained by Carter, and those obtained using the wellbore storage correction as presented in Eq. 21. Fig. 4 presents Carter's drawdown curves and the corrected curves for cases not involving non-Darcy flow. As can be seen from Table 2, use of the storage correction improves the estimate of flow capacity.

Inspection of Fig. 4 indicates that the correction tends to straighten the drawdown at times as short as 1/4 hour for Carter's cases.

\(^*\) Strictly speaking, \( dp_{w}/dt \) is not an accurate representation of the change in volume of gas in the wellbore for a flowing well. Because of the frictional pressure drop in the tubing, the average pressure for gas in the tubing should be evaluated separately from the average pressure of the static gas column in the annular space. Although adequate information is available to calculate the pressure-depth relationship for both columns, it is often reasonably accurate to use the bottom-hole rate of pressure change corrected to the mean depth of the well.
Inspection of Table 2 also indicates an interesting trend. The corrected flow capacities for Carter's cases including the effect of non-Darcy flow are not as close to the actual capacity as those cases which do not include a non-Darcy flow effect. Note also that the estimate of flow capacity appears to become poorer as flow rate increases when non-Darcy flow is present.

**EFFECT OF NON-DARCY FLOW ON GAS-WELL TESTING**

Smith, and Swift and Kiel' have presented significant analyses of gas-well testing including the effect of non-Darcy flow. Basically, their results indicate that non-Darcy flow leads to an additional pressure drop near the wellbore which can be treated as a flow-rate dependent skin effect. That is, the skin effect normally calculated for either build-up or down-draw tests performed long enough to avoid wellbore storage effects is actually an effective skin effect which includes components due to wellbore damage or stimulation, effects of perforations, partial penetration and, finally, non-Darcy flow. Swift and Kiel presented one method to obtain the non-Darcy flow coefficient from build-up tests. An alternate procedure that can be used for both drawdown and build-up testing is to compute the effective skin effect \( S' \) from long-time build-up or drawdown test data and the following equations normally used for build-up and drawdown testing.

For build-up, liquid flow analogy:

\[
S' = 1.151 \left( \frac{p_{\text{d}} - p_{\text{wf}}}{m} \right) + 3.23
\]

or using the gas flow analog,

\[
S' = 1.151 \left( \frac{p_{\text{g}} - p_{\text{w}}}{m'} \right) + 3.23
\]

For drawdown, the same equations may be used if \( p_i \) is substituted for \( p_{\text{d}} \), and \( p_{\text{wf}} \) is substituted for \( p_{\text{wf}} \). Note that Eqs. 27 and 28 are based on the assumption that flow time before shut-in is much larger than the build-up time of one hour. This is usually a valid assumption, but care should be exercised when these equations are applied to drill-stem testing, or any build-up test involving a very short production period.

The effective skin effect \( S' \) should be computed for at least two different drawdown and/or build-up tests at two different constant flow rates, \( q_i \) and \( q_o \), because:

\[
S' = S + Dq
\]

a plot of \( S' \) vs \( q \) should result in a straight line. The slope of the line is \( D \), the non-Darcy flow constant, and \( S \) is the true skin effect which may be found by extrapolating to zero flow rate (\( q = 0 \)). The true skin effect \( S \) contains the effect of all resistances to flow near the wellbore other than non-Darcy flow. On occasion, it is useful to separate \( S \) into various components. For example, \( S \) can be considered to be the sum of various skin effects due to altered permeability near the wellbore, partial penetration and perforations. The portion of the skin effect resulting from partial penetration and perforations may be estimated from information published by Brons and Martin. That is,

\[
S = S_i + S_{\text{cor}} + S_{\text{stor}}
\]

Through use of Eq. 30, it is possible to estimate the result of specific stimulation operations.

Eq. 29 and the method described to determine the non-Darcy flow coefficient have led to an important observation in tests of gas wells. On two separate occasions, gas-well build-up tests have indicated moderate damage at the flow rates employed in the build-up tests. However, analysis using Eq. 29 indicated the true skin effect to be negative. The effect of non-Darcy flow can sometimes cause a severe pressure drop which appears to be wellbore damage, and may actually obscure the effect of fractures near the wellbore. Misinterpretation of this result could lead to unnecessary, ineffective and costly well stimulation efforts.

Should the effective skin effect \( S' \) be constant for build-ups or drawdowns at two different flow rates, it is indicated that non-Darcy flow is not important at the highest test flow rate used. This does not mean that non-Darcy flow may not be important at a higher test-flow rate. In this event, the magnitude of the non-Darcy flow constant can be estimated from information published by Tek, Coats and Katz. Comparison of the Swift and Kiel and Tek, Coats, and Katz results indicates:

\[
D = 2.715 \times 10^{-6} B M_{\text{p}} k /
\]

where

\[
B_i = 5.5 \times 10^{12} k^4 \left( \phi S_o \right)^{3/4}
\]

Comparison of values of non-Darcy flow constants estimated from Eq. 31 with those obtained from well-test data has indicated estimated values may be in error by as much as 100 per cent. However, estimates using Eq. 31 have been useful in determining whether non-Darcy flow was likely to be important at planned gas-well producing rates higher than test rates. Another indication of the import-

---

**TABLE 2—COMPARISON OF FLOW CAPACITY DETERMINATIONS FROM CARTER'S DATA, Ref. 10**

<table>
<thead>
<tr>
<th>Solution</th>
<th>( q ) Mscf/D</th>
<th>( k ) (md-ft)</th>
<th>( S ) (md-ft)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1600</td>
<td>45.6</td>
<td>52.7</td>
<td>Non-Darcy flow effect present with storage</td>
</tr>
<tr>
<td>4</td>
<td>3200</td>
<td>35.8</td>
<td>48.5</td>
<td>Non-Darcy flow effect present with storage</td>
</tr>
<tr>
<td>7</td>
<td>1600</td>
<td>47.9</td>
<td>52.7</td>
<td>Darcy flow with storage effect only</td>
</tr>
<tr>
<td>8</td>
<td>3200</td>
<td>47.8</td>
<td>50.0</td>
<td>Darcy flow with storage effect only</td>
</tr>
</tbody>
</table>

---

**FIG. 4—APPLICATION OF WELLBORE STORAGE CORRECTION TO CARTER'S COMPUTED GAS-WELL DRAWDOWN DATA (Ref. 10).**
gence of non-Darcy flow is an isochronal deliverability-
curve slope less than unity ($n < 1$).

In summary, the non-Darcy flow coefficient $D$ can be
determined easily from either build-up or drawdown data
at two or more different rates using the usual skin-effect
equation (Eqs. 27 and 28) and Eq. 29.

Let us now consider the combined effect of non-Darcy
flow and wellbore storage. After Smith,2 and Swift and
Kiel,3 the proper relation for drawdown for constant-rate
production including non-Darcy flow becomes:

$$\frac{k\hbar(p_s - p_w)}{141.4 q g B} = \frac{p_d(t_o)}{1 + S + D q} \ldots (33)$$

Again, liquid-flow nomenclature has been used to simplify
the presentation. If we use the van Everdingen1 and Hurs:"expression
for the approximate sand-face flow rate generated
by wellbore storage effects, Eq. 15, and employ the
principle of superposition, we can write an equation analo-
ous to Eq. 16 which includes the effect of non-Darcy
flow:

$$p_u(t_o) = p_d(t_o) + S + D'(1-e^{-Bt})$$

$$-\frac{1}{2} e^{-Bt} - 1 - n \beta - 2 y + 1 + 4 e^{B t} + E i(B t_o)$$

$$+ 2S + 2D'(1-e^{-Bt}), \ldots \ldots \ldots (34)$$

where

$$\Xi = \frac{k\hbar(p_s - p_w)}{141.4 q g B} = \frac{19.87 \times 10^5 k h T_o (p_s^2 - p_w^2)}{q g B} \ldots \ldots \ldots (35)$$

and

$$D' = D q \ldots \ldots \ldots \ldots (36)$$

Thus $D'$ is a dimensionless drop caused by non-Darcy
flow and has significance similar to the skin effect $S$. Note
however, that $D'$ may not have values less than zero. Oc-
casionally, negative values may arise in working with field
data. This usually appears to be a result of poor flowing
pressures.

The additional pressure drop caused by non-Darcy flow
near the wellbore can be estimated in a fashion similar
to that described for the skin effect by Matthews.4 That is,

$$\Delta P$$

for non-Darcy flow, psi = 0.87 $m D' \ldots \ldots (37)$$

or

$$\Delta P$$

for non-Darcy flow, psi = 0.87 $m D' \ldots \ldots (38)$$

When non-Darcy flow is important, this additional pres-
sure drop is a natural consequence of gas flow, and should
not be removed in calculation of flow efficiencies.

Eq. 34 can be used to produce pressure drawdown
curves like those shown on Fig. 3, but such curves do not
appear useful. Much can be learned by inspection of 
Eq. 34, however. It is clear that at long times, the result
approaches Eq. 33. Thus, even in the presence of wellbore
storage effects, the non-Darcy flow effect can be evalu-
ated as suggested previously if the drawdown or build-up
tests are run long enough. Eq. 34 can also be used to pro-
duce a wellbore storage correction equation similar to
Eq. 20. The result is:

$$(p_s - p_w)(q/q_s) = \frac{m}{2.303} \left[ \ln \frac{0.00634 \dot{k} t}{\phi r_s} + 0.80907 + 2S + 2D'(q_s^2/q) \right] \ldots \ldots (39)$$

Eq. 39 may be used for gas flow using Matthews's liquid
flow analogy, or we can write a similar equation for gas
flow as:

$$(p_i - p_w)(q/q_s) = \frac{m'}{2.303} \left[ \ln \frac{0.00634 \dot{k} t}{\phi r_s} + 0.80907 + 2S + 2D'(q_s^2/q) \right] \ldots \ldots (40)$$

Pressure build-up can be handled similarly to Eq. 18,
except the term $2D'(q_s^2/q)$ must be added to the skin
effect within brackets.

Eq. 39 has a great deal of significance. First, inspection
of Eq. 39 reveals that the previous correction for wellbore
storage presented by Eq. 20 may fail if non-Darcy flow is
important. Because we often do not know the impor-
tance of non-Darcy flow before a field test, it would ap-
pear that Eq. 39 or 40 should be used often. At first
 glance, this appears a formidable task. However, all of
the terms on the right in both Eqs. 39 and 40 may be
lumped into a constant except time-dependent terms and
$m$ or $m'$, and $D'$. Thus Eq. 39 can be written:

$$(p_i - p_w)(q/q_s) = m \left[ \ln \frac{t + C + 0.87 D'(q_s^2/q)}{2} \right] \ldots \ldots (41)$$

and Eq. 40 can be written:

$$(p_i - p_w)(q/q_s) = m' \left[ \ln \frac{t + C' + 0.87 D'(q_s^2/q)}{2} \right] \ldots \ldots (42)$$

The unknowns in Eq. 41 are $m$, $C$, and $D'$. Pressure draw-
down data at three flowing times permits solving three
simultaneous equations for $m$, $C$ and $D'$. Thus a single
drawdown test may be used to obtain both the normal
slope, $m$, and formation flow capacity, and the non-Darcy
flow constant. This results by virtue of the fact that well-
bore storage causes a changing sand-face flow rate. This
result can be generalized. Any well test which results in
a changing sand-face flow rate can, at least in principle, be
analyzed to yield both the skin effect and the non-Darcy
flow constant.

We can inspect the utility of the preceding by reana-
lzing Carter's5 computed drawdown data by means of
Eq. 42. Drawdown data at 0.4, 1 and 4 hours were used
to compute $m'$, $C'$ and $D'$. Table 3 presents a com-
parison of the flow capacity determinations made by Carter
and those obtained from slopes from the analysis indicated by
Eq. 42. Values of the non-Darcy flow constant determined
by simultaneous solution are also shown.

It is apparent from Table 3 that both the formation
flow capacity and the non-Darcy flow constant can be
determined with reasonable accuracy from a single flow
test for Carter's cases. A sample calculation is presented
in the Appendix.

**DISCUSSION**

Although much of the preceding development has been
presented both in terms of liquid and gas flow, the following will discuss features of applications to gas-well testing. These appear to be most significant.

Both wellbore storage and non-Darcy flow effects near the wellbore affect the early portion of either a drawdown or a build-up test on a gas well. The duration of wellbore storage effects can be estimated approximately from Fig. 2 or Eq. 11, developed from van Everdingen and Hurst's studies. If drawdown or build-up tests are run long enough after wellbore storage effects are insignificant to determine the slope of the normal type of pressure-log time plot, formation flow capacity can be determined in the usual manner. If non-Darcy flow effects are present, the skin effect determined from a long-time drawdown or build-up test will be an effective skin effect. It will include an apparent skin due to non-Darcy flow. To determine the portion of the effective skin due to non-Darcy flow from long-time tests, it is necessary to perform two or more tests at different flow rates. This procedure usually affords the best information. Quite often, a normal isochronal deliverability test will provide sufficient information.

Wellbore storage effects usually do not persist long for gas wells. Nevertheless, the modern trend toward short drawdown (deliverability) testing makes consideration of wellbore storage important in many cases. If non-Darcy flow is not important, short-time drawdown data can be corrected approximately for wellbore storage using an extension of the Gladfelter, Tracy and Wilsey method for oilwell build-up testing. This correction straightens the early portion of a gas-well build-up or drawdown and permits better estimates of the formation flow capacity. If non-Darcy flow is important, the presence of wellbore storage effects in short-time tests permits a new method of analysis which not only corrects the data for wellbore storage, but also yields the non-Darcy flow constant from a single test. This method should be of considerable use in re-analyzing old data for non-Darcy flow effects. If wellbore storage is not important, it is necessary to run tests long enough to be certain the proper straight line has been reached, and find the non-Darcy effect from tests at two or more flow rates.

CONCLUSIONS

Both non-Darcy flow and wellbore storage effects can distort the early portion of either drawdown or build-up tests for gas wells. If both effects are significant, it is possible to estimate the flow capacity and non-Darcy flow constant from a single flow test. It is likely that the combination of non-Darcy flow and wellbore storage is an important factor in short-time gas-well testing which can lead to variations in flow capacity estimates.

Non-Darcy flow can be treated as a flow-rate dependent skin effect. With this concept, it is possible to determine the portion of the effective skin effect resulting from non-Darcy flow, if tests are available at two or more flow rates. These tests should be run long enough to ensure that wellbore storage effects are negligible. This approach provides an alternate procedure to determine non-Darcy flow effects (to that of Swift and Kiel) which depends only upon a reinterpretation of the familiar skin effect of van Everdingen and Hurst.

Non-Darcy flow can cause a pressure drop near a gas wellbore serious enough to make a well completed in a fractured region appear to be damaged. Determination of the extent of non-Darcy flow can thus avoid unnecessary stimulation efforts when the apparent skin effect includes a large non-Darcy flow component.
proximate wellbore storage and non-Darcy flow. See Eq. 34.

\[ q = \text{surface production rate, STB/D for oil, Mscf/D for gas} \]
\[ q_0 = \text{wellbore unloading rate, STB/D or Mscf/D} \]
\[ q_s = \text{sand-face production rate, STB/D or Mscf/D} \]
\[ r_c = \text{wellbore radius, ft} \]
\[ S = \text{true skin effect, dimensionless} \]
\[ S' = \text{total effective skin effect (See Eq. 29), dimensionless} \]
\[ S_a = \text{gas saturation, fraction of pore volume} \]
\[ S_b = \text{skin effect caused by permeability alteration near wellbore} \]
\[ S_{app} = \text{apparent skin effect caused by partial penetration of formation} \]
\[ S_{per} = \text{apparent skin effect caused by perforated liner in well} \]
\[ t_f = \text{flowing time, days} \]
\[ t_p = \text{production prior to shut-in, days} \]
\[ T = \text{temperature, °R} \]
\[ T_w = \text{base temperature for standard gas volume measurement, °R} \]
\[ T_v = \text{average wellbore temperature, °R} \]
\[ V_a = \text{annular volume, cu ft ft of depth} \]
\[ V_w = \text{total wellbore volume (tubing and annulus), cu ft} \]
\[ \phi = \text{density, lb/cu ft} \]
\[ \phi = \text{porosity, fraction of bulk volume} \]
\[ \mu = \text{viscosity, cp} \]
\[ \gamma = \text{Euler's constant, 0.57722} \]
\[ Z = \text{gas-law deviation factor} \]
\[ Z_0 = \text{gas-law deviation factor at pressure } p \]
\[ Z_v = \text{gas-law deviation factor at temperature } T_v \]

REFERENCES


APPENDIX

THE GLADFELTER-TRACY-WILSEY CORRECTION FOR WELLBORE STORAGE

To explore the Gladfelter-Tracy-Wilsey correction for wellbore storage, we will use an expression presented by van Everdingen and Hurst. We assume that the effect of wellbore storage is to cause a sand-face flow rate described by:

\[ q_s = q(1 - e^{-eta t_p}) \]

We now employ the principle of superposition to find the dimensionless pressure drop, \( \bar{p}_o(t_0) \), resulting from the sand-face flow rate described by Eq. 15.

\[ \bar{p}_o(t_0) = \int_0^{t_0} (1 - e^{-\beta't'}) p'_0(t_0 - t') \, dt' + S(1 - e^{-\beta't_0}) \]

where,

\[ p'_0(t_0) = \frac{d}{dt} \left[ p_o(t_0) \right] \]

The dimensionless pressure drop can be approximated by the line-source solution:

\[ p_o(t_0) \approx \frac{1}{2} \left[ -\ln(1 - \frac{\pi t_0}{4}) \right] \]

Thus:

\[ p'_0(t_0) = \frac{e^{-\pi^2 t_0^2}}{2t_0} \]

Solution of Eqs. A-1 and A-4 was presented by van Everdingen and Hurst and is reproduced as Eq. 16 in the main text of this paper:

\[ \bar{p}_o(t) = p_o(t_0) + S - \frac{1}{2} e^{-\beta t} \left[ -\ln(1 - 2 + \frac{1}{2} \left( \beta t_0 + 25 \right) \right] \]

Now, let us use Eq. 16 to explore the Gladfelter, Tracy and Wilsey approximation for wellbore storage during pressure build-up. The Gladfelter, Tracy and Wilsey approximate expression for pressure build-up is given as Eq. 18:

\[ p_o(t_0) + S = k(t) = \frac{1.151}{141.4 \sqrt{p_o}} \]

From the definition of the dimensionless pressure drop, \( \bar{p}_o(t_0) \):

\[ \bar{p}_o(t_0) = k(t) \]

\[ p_o(t_0) = \frac{1.151}{141.4 \sqrt{p_o}} \]

\[ p_o(t_0) = \frac{1.151}{141.4 \sqrt{p_o}} \]
As shown by Gladfelter, Tracy and Wilsey in their Eq. 8, the pressure difference on the left-hand side of Eq. 18 can be expressed by superposition of pressure drawdowns as:

\[ (p_{w1} - p_{w2}) = (p_1 - p_2) - \left[ (p_1 - p_{w1}) + (p_2 - p_{w2}) \right] \]

where the subscript in brackets indicates a flowing well pressure for the time period in brackets.

By means of Eqs. A-5 and A-6 we can write:

\[ (p_{w1} - p_{w2}) = \frac{m}{1.151} \left[ p_o(t_{m0}) - p_o(t_{m1}) + \bar{p}_o(t_{m0}) + S \right] \]

The last dimensionless pressure drop includes the effect of wellbore storage, and this factor may be evaluated from Eq. 16. Values of \( \bar{p}_o(t) \) could have been used in Eq. A-7, but use of \( p_o(t) \) leads to a useful simplification. From Eq. 15, the \( q_{st} \) during pressure build-up can be written as:

\[ q_{st} = q e^{-\beta t_{st}} \]

Thus:

\[ \frac{q - q_{st}}{q} = (1 - e^{-\beta t_{st}}) \]

Finally, we substitute Eqs. A-7 and A-9 into Eq. 18 to yield:

\[ \bar{p}_o(t_{m0}) + S = (1 - e^{-\beta t_{m0}}) \left[ \frac{\ln 0.00634}{\phi \mu c r_w^6} + 0.80907 + 2S \right] + p_o(t_{m1}) - p_o(t_{m0}) \]

Eq. A-10, then, is just an alternate way to state the Gladfelter, Tracy and Wilsey approximation, Eq. 18. From Eq. VI-15 in van Everdingen and Hurst:

\[ p_o(t_{m0}) = \frac{1}{2} [ \ln t_o + 0.80907 ], \text{ for } t_o > 100 \]

and Eq. A-10 finally becomes:

\[ \bar{p}_o(t_{m0}) + S = (1 - e^{-\beta t_{m0}}) \left[ \frac{\ln 0.00634}{\phi \mu c r_w^6} + 0.80907 + 2S \right] + p_o(t_{m1}) - p_o(t_{m0}) \]

We can rearrange Eq. 16 by factoring \( p_o(t_{m0}) + S \) from the right side and write an alternate expression for \( p_o(t_{m0}) \):

\[ p_o(t_{m0}) + S = (p_o(t_{m0}) + S)(1 - e^{-\beta t_{m0}}) \left[ \frac{\ln \beta - 2 \gamma + \ln 4 + Ei(\beta t_{m0}) + 2S}{2(p_o(t_{m0}) + S)} \right] \]

We can now compare Eqs. A-12 and A-13 and determine under what circumstances they will be equivalent. If the producing time, \( t_o \), is much larger than the shut-in time, \( t_i \) (say 100 times as long), the last two terms of Eq. A-12 would be nearly equal and cancel each other. Since the logarithmic approximation for \( p_o(t) \) given by Eq. A-11 applies for dimensionless times greater than 100, this would mean that dimensionless producing times should be greater than about 10,000. It is clear that at long build-up times (but still small compared to \( t_i \), the exponential terms become negligible and Eqs. A-12 and A-13 become equivalent. Equivalency would also be assured if the quotient in brackets on the right of Eq. A-13 were unity. This can be shown to be true under certain conditions. If the product \( (\beta t_{m0}) \) is smaller than 0.02:

\[ E(\beta t_{m0}) \approx E(\beta t_{m0}) \approx - \ln \left[ \frac{1}{\beta t_{m0}} + \gamma \right] \]

Substitution of the approximation in Eq. A-14 and use of Eq. A-11 yields:

\[ - \ln \beta - 2 \gamma + \ln 4 + Ei(\beta t_{m0}) + 2S \approx 2[p_o(t_{m1}) + S] \]

for \( \beta t_{m0} < 0.02 \), and \( t_o > 100 \).

Basically, then, we have shown that the Gladfelter-Tracy-Wilsey result can be obtained from the van Everdingen and Hurst treatments of wellbore storage, under certain conditions. The important realization is that shown by Eq. A-15. We can use Eq. A-15 to produce a correction for wellbore storage on pressure drawdown which is similar to the Gladfelter-Tracy-Wilsey correction for pressure build-up. Substitution of Eq. A-15 in Eq. A-13 and changing the time parameter from \( t_{m1} \) to \( t_{m0} \), the flowing time, yields:

\[ \bar{p}_o(t_{m0}) + S = (1 - e^{-\beta t_{m0}}) \left[ p_o(t_{m0}) + S \right] \]

and substitution of Eq. A-15 yields:

\[ \bar{p}_o(t_{m0}) + S = q_{st} \left[ p_o(t_{m0}) + S \right] \]

Using Eq. A-5 to replace \( \bar{p}_o(t_{m0}) \), and Eq. A-11 to replace \( p_o(t_{m0}) \) yields the final result in engineering units presented as Eq. 20 in the main text:

\[ (p_1 - p_{w1})(q/a) = m \frac{\ln 0.00634}{2.303 \phi \mu c r_w^6} + 0.80907 + 2S \]

Thus, multiplication of the observed pressure drawdown by the ratio of the final stabilized rate to the sand-face rate actually existing at specific drawdown times should provide a linear relationship with the logarithm of drawdown time.

It is informative to check the use of Eq. A-12 for pressure build-up and Eq. A-16 for pressure drawdown to see whether a straight-line portion does indeed exist, and whether it will have the proper slope.
Effect, S. This verifies the conclusion for the dimensionless pressure drop including the non-Darcy flow coefficient: 

Eq. A-12 can be rearranged to:

\[ p_0(t_{n+1}) - S = \frac{p_0(t_n) - S}{1 - e^{-\beta \Delta t}} + p_0(t_n) \]

Remembering that Eqs. A-12 or A-18 are just alternate ways to state the Gladfelter-Tracy-Wilsey approximation. Eq. A-18, a plot of Eq. A-18 should have the same slope as the normal \( p_0(t_n) \) vs \( \log t_n \), or 1.15. Fig. 5 presents both \( p_0(t_n) \) and the \( p_0(t_n') \) obtained from Eq. A-18 for values of producing time, \( t_{n0} \), of 0.1 \( \beta \) of 10 \( \gamma \), and \( S \) of zero. As can be seen, the slope of the corrected dimensionless pressure drop is almost the same as the true dimensionless pressure drop over a large range in dimensionless time. The same result is achieved for values of the skin effect. S. This verifies the Gladfelter-Tracy-Wilsey statement that the correction yields correctly the slope of the straight line after the initial drop in corrected values.

At large times, the slope increases and values of formation flow capacity determined from the slope would be in error. Also at times below about \( 8 \times 10^{-3} \), the corrected pressure drops tend to rise rapidly because the denominator of Eq. A-18 approaches zero. The proper straight line can be identified as the first straight line after the initial drop in corrected values.

A similar test may be made for the pressure drawdown correction suggested in this paper by means of Eq. A-16. The results of such tests are similar to that described above for pressure build-up.

EFFECT OF NON-DARCY FLOW NEAR WELLBORE ON WELLBORE STORAGE CORRECTION

We use the concept that non-Darcy flow near a wellbore may be considered similar to a flow-rate dependent skin effect. This implies the assumption that steady-state flow exists within the non-Darcy flow region near the well. As in the previous section in this Appendix, we assume Eq. 15 adequately represents the sand-face flow rate, and employ the principal of superposition to write an expression for the dimensionless pressure drop including the effect of non-Darcy flow:

\[
\Xi = \int_{t_0}^{t} \left( 1 - e^{\beta t_n'} \right) p_0(t_n - t_n') dt_n' + S(1 - e^{\beta t_0})D' \left( 1 - e^{\beta t_n} \right)^2
\]

The solution of Eq. A-19 is presented as Eq. 34 in the main text follows readily from the work of van Everdingen' and Hurst.

DETERMINATION OF FORMATION FLOW CAPACITY AND NON-DARCY FLOW CONSTANT IN PRESENCE OF WELLBORE STORAGE EFFECTS

Given the drawdown and reservoir data below for a flow test on a gas well, determine: (1) the estimated duration of wellbore storage effects, and (2) the corrected \( m' \) and non-Darcy flow coefficient for this well. Reservoir and fluid data are given in Table 4.

**SOLUTION**

We may use either Eqs. 11 or 14 to estimate the duration of wellbore storage effects. Using Eq. 14:

\[
t = \frac{4.785 m' m'' L'}{k h}
\]

\[
t = \frac{(4.785) (0.0176) (0.000409) (425/\pi)}{(50)}
\]

\[= 0.093 \text{ days} = 2.24 \text{ hours.}
\]

This estimate was made using the initial values of gas properties rather than average properties at an average drawdown pressure. If a lower average pressure were used, the estimated time would increase. It is clear that wellbore storage may have an important effect on the initial portion of this test.

It is first necessary to estimate the sand-face flow rate for this test. We will assume that gas physical properties can be represented adequately by the initial values. From Eq. 2, we estimate the portion of the gas produced from...
TABLE 6

<table>
<thead>
<tr>
<th>(hr)</th>
<th>p&lt;br&gt; (psi)</th>
<th>p&lt;sub&gt;0&lt;/sub&gt; - p&lt;br&gt; psi'</th>
<th>q&lt;sub&gt;s&lt;/sub&gt; (Mscf/D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.232</td>
<td>1715</td>
<td>3.35</td>
<td>1.231</td>
</tr>
<tr>
<td>0.6</td>
<td>1328</td>
<td>3.50</td>
<td>1.177</td>
</tr>
<tr>
<td>2.2</td>
<td>968</td>
<td>4.47</td>
<td>1.041</td>
</tr>
</tbody>
</table>

the decrease of pressure in the wellbore:

\[ C_s \text{res cu ft/psi} = V \cdot c_i = (425)(0.000409) = 0.174 \]

or:

\[ \text{Mscf/psi} = \frac{q}{C_s} = \frac{C_s}{R_s} = (0.174)/(0.000585 \times 10^5) = 0.02965. \]

From Eqs. 3 and 4:

\[ q_s = q - q_0 = q + \left( \frac{C_s}{R_s} \right) \left( \frac{dp}{dt} \right) = 3,200 \]

\[ + 0.02965 \left( \frac{dp}{dt} \right) (24 \text{ hr/D}) \]

\[ q_s = 3,200 + 0.712 \left( \frac{dp}{dt} \right) \text{ psi/hr}. \]

The rate of pressure change can be computed from the given drawdown data, and the sand-face flow rate found from the above equation. Calculations are presented in Table 5.

It is informative to inspect the results of Table 5. As can be seen, the sand-face flow rate at 0.266 hours flowing time is only a little over half the surface flow rate of 3,200 Mscf/D. Note also that the sand-face flow rate is within 3 per cent of the surface flow rate by 2.25 hours. It was estimated that wellbore storage effects would last about 2.24 hours.

The normal drawdown slope \( m' \) and the non-Darcy flow constant may now be estimated by means of Eq. 42. Solution will be made using drawdown data at 0.232, 0.6, and 2 hours. Values of \( (q/q_s) \) from Table 5 were smoothed before solution. The data used are shown in Table 6.

Thus, the simultaneous equations we wish to solve are:

\[ (1.531)(2.35) \times 10^6 = m' \left[ \log_a (0.232) + C + 0.87D'/1.531 \right] \]

\[ (1.117)(3.5) \times 10^6 = m' \left[ \log_a (0.6) + C + 0.87D'/1.177 \right] \]

\[ (1.041)(4.47) \times 10^6 = m' \left[ \log_a (2.0) + C + 0.87D'/1.041 \right] \]

Simultaneous solution yields:

\[ m' = 0.836 \times 10^6 \]

\[ D' = 1.21 \]

Fig. 6 presents the pressure-squared difference vs log time normally plotted for gas-well drawdown data. Because of the curvature of the data, it would be extremely difficult to choose a straight line from these data. The dashed line was drawn arbitrarily on Fig. 6 to illustrate the slope \( m' \) determined from simultaneous solution. It is clear that much longer flowing time would be necessary to establish the straight line in the normal manner. Note also that the simultaneous solution was obtained from data for 0.232 hour to 2 hours only.

The above actually does not establish the validity of the slope \( m' \) obtained by simultaneous solution. However, the sample problem was taken from the Solution 4 by Carter, and the true \( kh \) and approximate \( D' \) values are known for this solution. The comparisons between the sample calculation and true values are quite good as was shown in Table 3 previously.