The Effect of Heat Transfer Between Nearby Layers on the Volume of the Steam Zones

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Summary
The work of Mandl and Volek is extended analytically to include the effect on the volume of the steam zone in one layer owing to heat transfer from a second layer undergoing steam injection. The contribution of the heat transfer delays the onset of heat convection across the advancing condensation front (the critical time) and significantly increases the volume of the steam zone even where 30 or more feet separate the layers. The increase in the heat content of the steam zone (or volume) increases with increasing ratio of the sensible to latent heat of the injected steam, and is larger than the increase in the total heat content. The interpretation for this behavior is that heat transfer delays the rate of condensation of the steam vapor, this being more important at poor steam qualities. Results of the simple approach presented here have been confirmed qualitatively through numerical simulation.

Introduction
In 1967, Mandl and Volek developed upper and lower bounds for the heat content and volume of the steam zone undergoing variable steam injection in a single layer. In 1981, Yortsos and Gavalas refined these bounds. This paper extends the work of Mandl and Volek to two nearby layers undergoing steam injection. The necessary information on the heat transfer between the two layers is taken from Prats. Because the volume of the steam zone is closely related to the amount of oil displaced, the results provide a simple approach that may be used to screen reservoirs with multiple layers for subsequent commercial steamflood evaluation.

Assumptions
The working equations developed by Mandl and Volek are based on the following assumptions:
• The temperature in the steam zone is that of the steam injected into the reservoir, which remains constant.
• All saturations, temperatures, and rock properties are uniform within the steam zone. Gravitational effects are neglected.
• The heated layer is of uniform thickness.
• The velocity of the condensation front is the same everywhere.
• Pressure gradients are considered to be negligible or nonimportant.
• The rates of injection of sensible and latent heat may vary with time.
• Thermal properties are uniform within and outside the layer undergoing steam injection.
• No heat is produced from the heated zones.

In considering the effects of heat transfer between nearby layers, all but one of the above assumptions are retained. This paper specifically considers constant injection rates of both sensible and latent heat. The manner in which the heat transfer between layers is handled is discussed with reference to Fig. 1. Steam is injected into a well open to two sands, Layers 1 and 2, separated by an impermeable center layer of nonzero thickness. First, consider steam injection only into Layer 1. This results in heat transfer by conduction into Layer 2. If we now inject steam into Layer 2, its heat content would be higher than normal by the amount of heat transferred from Layer 1.

Mandl and Volek developed the size of the steam zone based on the rate of heat lost to surrounding layers. An additional assumption is made here, that the rate of heat lost from a layer undergoing steam injection is offset by the total rate of heat transfer from an adjacent layer undergoing heat injection. With this one and sole additional assumption, the development parallels that of Mandl and Volek.

Solution Outline
The solution has two phases. In the first, the distribution of the total heat between the two layers is determined based on a method of Prats. The total heat content of Layer 1 under steam injection is determined as a function of time, taking into consideration the differences in the thermal properties of the other layer to be injected, the center layer, and the overburden and underburden. This is denoted by . The heat transferred by conduction from Layer 1 to Layer 2 is also determined as a function of time, and this is denoted by . In a similar manner, the total heat content in Layer 2 under steam injection is determined by , and that transferred by conduction from Layer 2 to Layer 1 by . Because the systems are linear, the heat contents during simultaneous steam injection are additive, so that the total heat content in layer (for ) is . The total rates of heat transfer from one layer to another are also obtained.

The second phase is the determination of the steam zone volume in each of the two layers. This is done following the work of Mandl and Volek and the refinements of Myhill and Stegemeier. Mandl and Volek introduced the concept of a critical time , at which heat is first transferred across an advancing steam condensation front. For , the steam zone is the entire heated zone. For , the steam zone is smaller than the equivalent volume of a heated zone at the steam temperature. The critical time is obtained based on the defined above.

The volume of the steam zone is controlled by the steam condensation rate, which is partly controlled by the rate of heat loss from the layers undergoing steam injection. Here, the rate of heat lost from a layer undergoing steam injection is offset by the total rate of heat transfer from an adjacent layer undergoing heat injection. Eq. A-9, an extension of Ref. 1 to account for the net rate of heat loss when steam is injected into a nearby layer, is the basis for the determination of the volume of the steam zone. Two important elements affecting the rate of heat transfer from the nearby layer are the rate of heat injected into that layer and the distance between the layers.

Somewhat expanded outlines of the procedures used are provided in Appendix A. Laplace transforms are used extensively in the development, with solutions in time obtained using the Stehfest inversion algorithm.

Results
For the constant rates of heat injection considered here, the most important parameters affecting heat transfer between two nearby layers are the properties of the intervening center layer: thickness, thermal conductivity, and volumetric heat capacity.

Base thermal conductivities and volumetric heat capacities for all layers are 35.0 Btu/ft°F-day and 38 Btu/ft³°F respectively. Thermal properties of the Center Layer were varied by approxi-
Fig. 1—Schematic diagram of the two-layer heat injection system.

Accordingly approximately ±10 percent, resulting in the data sets shown in Table 1. Four thicknesses are considered for the Center Layer: 10 (Base Case), 20, 30, and 40 ft, shown in Table 2.

For all results shown, the height of Layer 1 is 30 ft, and that of Layer 2 is 30 ft unless otherwise indicated. For the thermal property set P1, 1.0 year corresponds to a dimensionless time of 1.4952. The dimensionless time is defined in terms of the thermal properties of Layer 1.

Equal and Constant Rates of Steam Injection. Fig. 2 duplicates the results presented by Myhill and Stegemeier for injection into one layer, and also shows the effect of injecting steam at the same rate into a nearby layer. The heat efficiency of the steam zone is plotted vs. the logarithm of the dimensionless time based on the properties of Layer 1. The heat efficiency of the steam zone is independent of the total rate of heat injection into a layer or on the steam temperature. Property sets P1 and P1G1, referred to as Case P1G1, were used to generate these results. Because the injection rates are the same for both layers and the properties are the same everywhere (Set P1), the results are valid for either layer. For injection into one layer, the results are independent of that layer’s thickness. But for injection into two layers, or when the thermal properties are not the same everywhere, the steam-zone heat efficiency does depend on all properties.

As may be noted, the beneficial effects of heat interaction on the steam-zone heat efficiency are more pronounced the lower the $f_{\text{hvs}}$, the fractional latent heat content of the injected steam, but are significant for any value of $f_{\text{hvs}}$ at sufficiently large dimensionless times. Fig. 3 displays the same results (Case P1G1) for times up to 10 years. Here we note that the improvements in the steam-zone heat efficiency are significant for times and values of $f_{\text{hvs}}$ of interest. For the 10-ft separation between the layers, thermal interaction between them is significant before 1 year. It may be established from the results presented in Figs. 2 and 3 that the increase in the steam zone owing to heat interaction is 1.6 for $f_{\text{hvs}} = 0.0909$ may look anomalous, but it corresponds to a smooth monotonic increasing steam-zone heat content with time. The steam-zone heat content is obtained by multiplying the steam-zone heat efficiency by the cumulative heat injected, which is proportional to time.

Fig. 4 shows the improvement factor in the steam zone volume arising from heat transfer between layers, again for Case P1G1, and for equal injection rates into both layers. The improvement factor in the steam zone volume is the ratio of the steam zone volumes with and without heat transfer between layers. Results are shown plotted for values of the parameter $R_{\text{SSL}}$, which is the ratio of the sensible to latent heat of the injected steam, corresponding to the values of $f_{\text{hvs}}$ used in Figs. 2 and 3. The relationship between these two parameters is

$$R_{\text{SSL}} = 1/f_{\text{hvs}} - 1 = 1/h_0 = C_w \Delta T(f_{\text{hvs}}).$$  \hspace{1cm} (1)

As may be noted, $R_{\text{SSL}}$ is an inverse measure of the thermal quality of the steam parameter, equal to the reciprocal of the parameter $h_0$ used by Myhill and Stegemeier. It is noted that because steam always has a sensible heat component under practical conditions, $R_{\text{SSL}}$ cannot be zero except for the limiting case in which the steam and initial formation temperatures are equal, which is not realistic. Mathematically, the case $R_{\text{SSL}} = 0$ represents an infinite steam quality, for which the critical time is infinitely large. This means that there is no heat downstream of the advancing condensation front. In other words, all the heat is within the steam zone.

Fig. 5 plots the same results vs. $R_{\text{SSL}}$ for several values of time, as well as the improvement factor at critical time, $I_c$. It was noted that for values larger than about $I_c + 0.2$, the improvement factor is essentially linear with $R_{\text{SSL}}$. Over this range, the curves in Fig. 4 may be interpolated linearly for values of $R_{\text{SSL}}$ not shown in the figure, and this is the reason for introducing this parameter. Also shown in Fig. 5 is $R_c$, the ratio of the critical times with and without steam co-injection. Steam co-injection increases critical times, so that $R_c$ is larger than 1.

Fig. 4 shows that the volume of the steam zone is increased by at least 38% after 2 years of steam coinjection, for any value of $R_{\text{SSL}}$. In fact, the poorer the steam quality parameter, the higher the improvement in the steam zone volume. At 10 years of steam coinjection, the minimum improvement factor in the volume of the steam zone owing to heat interaction is 1.6 for $R_{\text{SSL}} = 0$, and is 4.6 for $R_{\text{SSL}} = 10.0$, the lowest value considered by Myhill and Stegemeier. For $R_{\text{SSL}} = 0$, the critical time is very large, and the steam zone volume coincides with the equivalent heated volume. That is, the heat content within the reservoir volume occupied by the steam zone equals the total heat content of the layer. Thus, the steam zone volume improvement factor is at least as large as the improvement factor in the total heat content.

Similar results, obtained for Cases P1G2, P1G3, and P1G4, are shown in Figs. 6 through 8, respectively. The thicker the intervening layer, the lower the improvement in the steam zone volume. But even for a 40-ft-thick center layer (Fig. 8), the improvement is no less than 32% after 6.5 years.

Of course, somewhat different results are obtained when using different thermal properties. These results are displayed in Fig. 9 for all data sets, and $R_{\text{SSL}} = 1.0$. For any thickness of the center

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**Table 1—Thermal Properties of Center Layer**

<table>
<thead>
<tr>
<th>Set</th>
<th>$\lambda_w$, Btu/ft°F.day</th>
<th>$M_w$, Btu/ft²°F</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>35.0</td>
<td>38.0</td>
</tr>
<tr>
<td>P2</td>
<td>31.8</td>
<td>34.5</td>
</tr>
<tr>
<td>P3</td>
<td>38.5</td>
<td>41.8</td>
</tr>
<tr>
<td>P4</td>
<td>31.8</td>
<td>41.8</td>
</tr>
<tr>
<td>P5</td>
<td>38.5</td>
<td>34.5</td>
</tr>
</tbody>
</table>

Note: Values of the parameters are obtained by multiplying or dividing the P1 Set values by 1.1.

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**Table 2—Thickness of Center Layer**

<table>
<thead>
<tr>
<th>Set</th>
<th>Thickness (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>10</td>
</tr>
<tr>
<td>G2</td>
<td>20</td>
</tr>
<tr>
<td>G3</td>
<td>30</td>
</tr>
<tr>
<td>G4</td>
<td>40</td>
</tr>
</tbody>
</table>
layer, results vary by less than about ±3% of the value for the Base Case. Similar results are obtained for other values of $R_{SSL}$. The range of thermal properties is relatively small compared to the possible variation in the thickness of the center layer and thus does not have a major impact on the improvement in the steam zone volumes.

**Different Constant Rates of Steam Injection.** Injection at different constant rates into two layers results in the steam heat efficiency of each layer depending on the rate ratio $Q_2/Q_1$. The heat efficiency of the layers cannot be equal for $Q_2/Q_1 \neq 1$. For $Q_2/Q_1 = 1.5$, Fig. 10 shows heat efficiencies for Case P1G1 and $R_{SSL} = 1.0$. For $Q_2/Q_1 = 1.5$, the rate of heat transfer from Layer 2 toward Layer 1 is 1.5 times that for equal rates. The rate of heat transfer from Layer 2 to Layer 1 is larger than in the other direction, which results in the heat efficiency of Layer 1 being larger than that of Layer 2. Also shown is the average heat efficiency of both layers, obtained using Eq. A-11. The average for equal rates, also shown, is slightly higher than for unequal rates. It can be proven that for thermal properties equal in every layer, and for Layers 1 and 2 of the same thickness, the maximum combined heat efficiency is obtained for equal rates of heat injection into each layer. This is true for any value of $R_{SSL}$.

Figure 10 also graphs the ratio of the steam zone areas, which for equal thickness layers equals the steam zone volume ratio. Whereas the heat efficiency of Layer 1 is larger than that of Layer 2, the steam area of Layer 1 is smaller than that of Layer 2. The area ratio changes with time, averaging approximately 85% over the 5 years shown in Fig. 10. The extension presented here to the model of Mandl and Volek\(^1\) tacitly assumes that the rate of heat

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**Fig. 2**—Steam-zone heat efficiency with and without heat interaction between layers plotted vs. dimensionless time for several values of the parameter $f_{av}$. Case P1G1.

**Fig. 3**—Steam-zone heat efficiency with and without heat interaction between layers plotted vs. real time for several values of the parameter $f_{av}$. Case P1G1.
loss from the steam zone in a layer undergoing steam injection is exactly reduced by the rate of heat transfer from a nearby layer, independent of the temperature distribution. When the steam temperatures in both nearby layers are about the same, which is likely in many field operations, this condition is best met when the heated zones in both layers extend about the same distance from the injection well. The more advanced the heated zone in one layer is relative to the other, the less applicable the results of this model to the less advanced layer, and the more applicable it is to the more advanced one.

A procedure for improving the estimate of the steam zone volume in the layer with the smaller heated area is discussed next.

This procedure is based on the distribution of the rate of heat loss from a layer undergoing constant heat injection, which is readily calculable (see Appendix B). As shown in Fig. 11, about 0.70 of the total rate of heat loss occurs from the 80% of the heated area closest to the injector over the range of dimensionless time from 0.01 to 100. The figure also displays results for 85, 90, and 95% of the heated area. The curves are essentially flat. The procedure consists of reducing the rate of heat transfer from the layer with the larger steam zone area by an amount based on the calculated steam zone area ratio of the two layers.

Fig. 12 shows the steam zone volume improvement factors for Layers 1 and 2 of Fig. 10 and their average. With regards to Fig. 10, at 5 years the ratio of the areas is about 71%, with an average

Fig. 4—Improvement factor in the size of the steam zone resulting from heat transfer between layers plotted vs. time for several values of the parameter $R_{RhSL}$. Case P1G1.

Fig. 5—Improvement factor in the steam zone volume resulting from heat transfer between layers plotted vs. parameter $R_{RhSL}$ for several values of time. $I_c$ is the improvement factor at critical time. $R_{tc}$ is the critical time ratio with and without steam coinjection. Case P1G1.
of about 80%. From Fig. 11, the improvement in the steam zone volume of Layer 1 should be reduced by about 57%. Applying this correction over the 5 years gives the adjusted improvement factor for Layer 1 and for the average of the layers shown in Fig. 12. Whereas the improvement factor for Layer 1 is reduced from 2.35 to 1.77 at 5 years, the average is only reduced from 1.91 to 1.67. The adjusted improvement factors are still significant.

The injected steam quality may also be different in the two layers. Fig. 13 shows the effect of choosing different values for $R_{\text{hsl}}$ for Case P1G1. This is equivalent to different rates of latent heat injection into the layers while maintaining the same total rate of heat injection into each. Because the injected steam into dual layers is likely to be wetter into the bottom layer, we choose $R_{\text{hsl}}$ in Layer 2 to be larger than that into Layer 1. Fig. 13 shows results for $R_{\text{hsl}} = 0.75$ in Layer 1 and 1.25 in Layer 2, and for $R_{\text{hsl}} = 1$ in each layer. Even though the average $R_{\text{hsl}} = 1.0$ in both cases, the size of the steam zone is slightly lower when the $R_{\text{hsl}}$ are different. For the conditions chosen, the steam zone areas are essentially equal, and no adjustment is warranted.

**Discussion**

The fact that the improvement in the steam zone volume resulting from coinjection generally is larger than that for the total heat content was not anticipated, but can be explained with some analyses. Results have been confirmed qualitatively through numerical simulation.

Because the simple model does not consider changes in injectivity with time, exact agreement with simulated results could
never be expected. Simulations were done at constant rate of water and enthalpy input, with pressure and temperature, and thus varying less than 10% from their average value. Also, the thermal properties of the layers are not independent of fluid saturations and temperature, which is the case in the simple model. Ten gridblocks were used between injection and producing wells, which affects the accuracy with which the steam front may be located. As a specific example, for Case P1G1 and an average value of $R_{SL} = 0.72$ over a 5-year period, an analysis of the simulated distributions of the vapor and hydrocarbon phase saturations and of the temperature indicates an approximate steam zone improvement factor of 1.8. This compares with an approximate value of 1.7 at 5 years interpolated from Fig. 4. Similar agreement was obtained for a few other values of $R_{SL}$.

Simulator runs were also made for dissimilar steam injection rates into the two layers, with results similar to those obtained by the simple model. In numerical simulation studies, it is tempting to simplify as much as possible, and simplifications are usually warranted. Where coinjection is to be evaluated, reducing the problem to injection into one layer at a time and adding results would underestimate the steam zone volumes appreciably, with corresponding results for oil production. Some simplified simulation studies done by the industry have considered the temperature distribution arising from injection into the other layer, and include that result as an initial condition for the other layer. This approach may be better than assuming no interaction, but it is arbitrary as to how to choose a representative initial condition. Where coinjection is to be evalu-
ated, our results indicate the need to simulate coinjection into both layers in order to assure meaningful results.

As to why the improvement in the steam zone volume generally is greater than that of the total heat content, the main reason is that the heat in the steam zone is lower than the total in the layer, and the improvement factors start from a different base reference. A second reason is that the increase in the amount of heat in the steam zone resulting from coinjection increases with increasing $R_{hSL}$. These two factors explain why the improvement in the steam zone volume is generally greater than that of the total heat content, and in fact why it increases with increasing $R_{hSL}$.

As is well known, the steam zone volume is closely connected to the amount of oil displaced. It is not intended that the results presented here be misused to predict an oil-recovery stream, as has been done by many to the procedure presented by Myhill and Stegemeier. It is considered likely that the procedures presented here would be most useful as a screening tool in identifying potential steamflood targets where there are several thin sands, none of which can support a commercial operation by itself. If the calculated improvement factor with coinjection is sufficiently attractive, further analyses would be necessary before implementing the steam injection operation. Another application is to help understand higher-than-anticipated oil production from ongoing steamfloods in which the separation between layers had been considered too large to have a significant effect. For those who prefer to use only

Fig. 10—Steam-zone heat efficiency vs. time for $R_{hSL}=1$ and $\frac{\dot{Q}_2}{\dot{Q}_1}=1.5$, Case P1G1. The average of the two layers is slightly lower when the rates are dissimilar. Note that the effective heated areas are different when the rates are not equal.

Fig. 11—Fraction of the rate of heat lost from the fraction $x$ of the equivalent heated area closest to the injection well as a function of dimensionless time.
numerical simulation in their evaluations, our results indicate all layers undergoing steam injection should be included even when the distance between layers is relatively large.

**Results and Conclusions**

Within the framework of the assumptions made, and of the results obtained, it is concluded that in the process of co-injecting steam into several layers from the same well:

1. Heat transfer between the layers may result in significant increases in the size of the steam zones, with anticipated corresponding increases in oil production.
2. The increase in the heat content of the steam zone owing to heat transfer exceeds the increase in total heat.
3. The increase in the steam zone volume increases with the ratio of the sensible to latent heat ratio of the injected steam.
4. Calculated steam zone volumes will be undervalued if heat transfer effects are not taken into account.
5. Heat transfer effects can be significant within a few years, even where 30 or more feet separate the layers.
6. Coinjection may identify a project for further evaluation, whereas steam injection only into any one sand does not.
Nomenclature

\[ A_{jN} = \frac{\lambda_y Y_y \cosh(\gamma_j \Delta h_j) - R_{jN}}{\theta_j M_j \Delta h_j \sinh(\gamma_j \Delta h_j)} + \frac{\lambda_y Y_y}{\theta_j M_j \Delta h_j} \]

\[ B_{jN} = \frac{M_j R_j N_j \epsilon + 1}{\gamma_j M_j \Delta h_j \sinh(\gamma_j \Delta h_j)} \]

\[ C_0 = \sqrt{(1 - 1) - (1 - \epsilon a)^2 + \sqrt{\pi}} \]

\[ C_1 = 2(\sqrt{1 - 1} - (1 - \epsilon a)^2) \]

\[ C_2 = -[\sqrt{1 - 1} - (1 - \epsilon a)^2 + 1] \]

\[ \text{erf}(x) = e^x \text{erfc}(\sqrt{x}), \text{dimensionless} \]

\[ f_{hw} = \text{fraction of the heat injection rate present as latent heat.} \]

\[ h_j = \text{vertical distances defined in Fig. 1, ft} \]

\[ H = \text{upper bound to the heat content of the steam zone, Btu} \]

\[ I_k = \text{steam zone volume improvement ratio at critical time, dimensionless} \]

\[ L_n = \text{latent heat of vaporization, Btu/lbm} \]

\[ M_j = \text{Volumetric heat capacity of Layer } j, \text{ Btu/ft}^3{\cdot}{}^\circ \text{F} \]

\[ M_{j*} = \theta_j \Delta S_j / \Delta T_j, (j = 1 \text{ or } 2), \text{ Btu/ft}^3{\cdot}{}^\circ \text{F} \]

\[ Q = \text{cumulative heat injected, Btu} \]

\[ R_{j,N} = \text{ratio of sensible to latent heat in steam, dimensionless} \]

\[ R_{j,N} = \left[ \frac{\beta_{Cj}}{\theta_j} \text{cosh}(\gamma_j \Delta h_j) \right] \]

\[ R_{j,j} = \text{cosh}^2(\gamma_j \Delta h_j) / \left[ \beta_{Cj} + \text{cosh}(\gamma_j \Delta h_j) \right] \]

\[ S_j = \text{saturation, dimensionless} \]

\[ t = \text{time, days} \]

\[ t_D = \theta_j \Delta T_j, \text{dimensionless time} \]

\[ u = \text{Laplacian transform variable, dimensionless} \]

\[ U(x) = \text{unit function, 1 for } x > 0, 0 \text{ for } x < 0 \]

\[ \sigma_j = \text{thermal diffusivity, ft}^2{\cdot}{}^\circ \text{F/day} \]

\[ \beta_{C,j} = \lambda_j \gamma_j / (\lambda_j \gamma_j), \text{dimensionless} \]

\[ \gamma_j = \theta_j \Delta S_j / \Delta h_j, \text{ft}^{-1} \]

\[ \Delta S_j = \Delta S_j / \Delta T_j, j = 1 \text{ or } 2, \text{ dimensionless} \]

\[ \Delta h_1 = h_1, \text{ thickness of Layer 1, ft} \]

\[ \Delta h_2 = h_2 - h_1, \text{ thickness of Layer 2, ft} \]

\[ \Delta h_C = h_C - h_1, \text{ thickness of Center Layer, ft} \]

\[ \Delta H_j = \text{correction to upper bound of the heat in the steam zone after critical time, Btu} \]

\[ \Delta T_j = \text{temperature of steam above reservoir temperature, } ^\circ \text{F} \]

\[ \theta_1 = \left( \sqrt{M_1 \Delta h_1} + \sqrt{M_2 \Delta h_2} \right) / M_1 \Delta h_1, \text{ days}^{-1/2} \]

\[ \theta_2 = \left( \sqrt{M_1 \Delta h_1} + \sqrt{M_2 \Delta h_2} \right) / M_2 \Delta h_2, \text{ days}^{-1/2} \]

\[ \lambda_j = M_j \alpha_j, \text{ thermal conductivity, Btu/ft}^2{\cdot}{}^\circ \text{F-day} \]

\[ \phi_j = \text{porosity, dimensionless} \]

Superscripts

\[ * = \text{Laplace transform} \]

\[ \cdot = \text{rate of change of } f \text{ with respect to dimensionless time} \]

References


Appendix A—Critical Time

For variable heat injection into only one layer, the critical time is defined by Mandl and Volek by the condition

\[ H(t) = Q(t), \quad \text{.......................... (A-1)} \]

It also may be found from expressions in Laplace space as the time at which

\[ L^{-1}(H(t) - Q(t)) = 0, \quad \text{.......................... (A-2)} \]

With the same procedures applied to two thermally interacting layers, the critical time in Layer \( j \) would be the time at which

\[ L^{-1}(H(t) - Q_j(t)) = 0, \quad \text{.......................... (A-3)} \]

where, from Prats,

\[ H_j(t) = H_j^1(t) + H_j^2(t) \]

\[ H_j^1(t) = \frac{\dot{Q}_j(t)}{u + A_{j,2,0}} + B_{j,2,0} \dot{Q}_j(t) \quad \text{.......................... (A-4)} \]

and

\[ H_j^2(t) = \frac{\dot{Q}_j(t)}{u + A_{j,2,0}} + B_{j,2,1,0} \dot{Q}_j(t) \quad \text{.......................... (A-5)} \]

Symbols are defined in the Nomenclature.

For constant rate of heat injection only into Layer 1, and all layers outside Layer 1 having the same thermal properties, Eq. A-4 reduces, as it should, to

\[ H_j(t) = \frac{\dot{Q}_j}{u + \sqrt{u}} \quad \text{.......................... (A-6)} \]

which for constant rate of heat injection results in

\[ H(t) = \left[ 2 \sqrt{\frac{\tau}{\pi}} - 1 + e^\tau \text{erfc}(\sqrt{\frac{\tau}{2}}) \right] \dot{Q}_j, \quad \text{.......................... (A-7)} \]

Heat Content of the Steam Zone—Upper Bound

Eqs. A-4 and A-5 give the upper bounds for the heat content of the steam zones for \( t > t_c \).

Heat Content of the Steam—Lower Bound

Eq. 41 of Mandl and Volek is the integral equation, valid for \( \tau = t_{1/2} - t_p > 0, \) used to determine the lower bound of the heat content in the steam zone. Here we consider constant rates of heat injection only. It can be solved in Laplace space, giving, in our notation,

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\[ \Delta H^a(u) = (\sqrt{1 - 1}) - \frac{\dot{Q}_i}{u} - H^a(u) \] ........................................ (A-8)

For injection into two layers, the same procedure is used to obtain the correction to the heat content of the steam zone:

\[ \Delta H^b(u) = (\sqrt{1 - 1}) - \frac{\dot{Q}_i}{u} - H^b(u) \] ........................................ (A-9)

With the weighting factor \(1 - f_{jw}\), used by Myhill and Stegemeier,\(^6\) the heat in the steam zone expressed as a fraction of the heat injected, known as the heat efficiency of the steam zone, is

\[ E_{jw}(t_d) = \frac{H_j(t_d) - (1 - f_{jw})\Delta H_j(t_d)}{\dot{Q}_i t_d} \] ........................................ (A-10)

The combined heat efficiency of both steam zones is then

\[ E_{jw}(t_d) = [E_{jw}(t_d) + H_{jw}(t_d) + H_{jw}(t_d)]/(\dot{Q}_i + \dot{Q}_i) \] ........................................ (A-11)

Special case: Injection only into Layer 1, and all layers outside Layer 1 having the same thermal properties. This is the case considered by Mandl and Volek in their Eq. 41. Making use of Eq. A-6 and dispensing with layer subscript 1 reduces Eq. A-9 to

\[ \Delta H^a(u) = (\sqrt{1 - 1}) - \frac{\dot{Q}_i}{u} - 1/(\sqrt{u} + 1) \] ........................................ (A-12)

In the time domain \(t\), the lower bound for the heat content of the steam zone per unit rate of heat injection is

\[ \Delta H(t_d) = U(t_d - t_d)(\delta H(t_d) - \delta H(t_d)) \] ........................................ (A-13)

where

\[ \delta H(t_d) = L^1[\Delta H(u)/\dot{Q}] \] ........................................ (A-14)

Although it has been customary to approximate this last equation,\(^7\) it can be solved to give

\[ \delta H(t_d) = C_0 + C_1\sqrt{t_d} + \text{erf}(\sqrt{t_d}) + C_2 \text{erf}(\sqrt{t_d}), \] ........................................ (A-15)

so that Eq. A-10 becomes

\[ E_{jw}(t_d) = H(t_d)/(\dot{Q}_i) \]

\[ - (1 - f_{jw})U(t_d - t_d)/[\delta H(t_d) - \delta H(t_d)]/t_d \] ........................................ (A-16)

**Appendix B—Rate of Heat Loss Along Heated Zone**

For constant rate of heat injection into one layer, the fraction of the total rate of heat lost from a fraction \(x\) next to the injector of the total heated area is

\[ f = \int_{t_d}^{\infty} Y(t_d, u) du / \int_{t_d}^{\infty} Y(t_d, u) du, \] ........................................ (B-1)

where

\[ Y(t_d, u) = \text{erf}(u)/\sqrt{t_d - u} \] ........................................ (B-2)

and \(t_d\) is the time at which

\[ H_1(t_d) = xH_1(t_d) \] ........................................ (B-3)

**SI Metric Conversion Factors**

- Btu/ft\(^2\)\(^\circ\)F × 6.706 611 = E401 = kJ/m\(^3\)\(^\circ\)K
- Btu/ft\(^2\)\(^\circ\)F\(\cdot\)day × 4.153 754 = E401 = W/m\(^\circ\)K
- ft × 3.048\(^*\) = E-01 = m
- year × 3.6525 = E402 = day

*Conversion factor is exact.*

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