Verification of Decline Curve Analysis Models for Production Prediction
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Abstract
Reserves and production in petroleum reservoirs can be estimated using empirical and analytical decline curve analysis models. There have been many such models. In this study, three representative models (exponential, harmonic, and the mechanistic Li-Horne models) were chosen to predict and match oil and gas production from both core samples and reservoirs with different permeability. The recovery and reserves were estimated using the three models and the results were compared. The comparison demonstrated that the recovery at an assumed economic limit predicted using the Li-Horne model was greater than the exponential model but was less than the harmonic model. Note that the exponential model tends to underestimate reserves and production rates while the harmonic model has a tendency to overpredict the reservoir performance. It was also found that the reserves predicted using the harmonic model were greater than one pore volume in some cases, which is physically impossible. The model predictions using the experimental data of recovery in the core samples were also compared to the true values. The results demonstrated that the Li-Horne model had the best estimation of the recoverable recovery compared to the exponential and the harmonic models in the cases studied.

Introduction
Several approaches can be used to estimate reserves and predict production in reservoirs. Numerical simulation techniques may facilitate the prediction of oil production but may also fail because of the great uncertainty and the complexity of reservoirs. Another frequently used technique is the decline curve analysis method. However many of the existing decline curve analysis models are heuristic and are based on the empirical Arps equations: exponential, hyperbolic, and harmonic models. The exponential decline curve tends to underestimate reserves and production rates; the harmonic decline curve has a tendency to overpredict the reservoir performance (Agbi and Ng). In some cases, as pointed out by Camacho and Raghavan, production decline data follow neither the exponential nor the harmonic model but cross over the entire set of curves.

Fetkovich provided a theoretical basis for the Arps equation in the case of exponential decline. The assumptions made were as follows: (1) single-phase flow; (2) no water injection and no water production; (3) production rate is proportional to \( \bar{p} - p_{w} \) \( (\bar{p} \text{ is the average pressure in the reservoir and } p_{w} \text{ the bottom pressure in the production well}) \). However two-phase flow and water production often occur in reservoirs developed by water flooding, especially at the later period of production. In this case, relative permeability and capillary pressure should be honored in a production prediction model with a theoretical basis. Note that the exponential and harmonic models often work better at the later period of production than during the early period.

Li and Horne proposed a mechanistic decline model based on previous theoretical and experimental studies (Li and Horne). The model reveals a linear relationship between the oil production rate and the reciprocal of the oil recovery or the cumulative oil production. Two-phase flow properties such as relative permeability and capillary pressure are included in this model. Li and Horne showed that the mechanistic model worked satisfactorily in different reservoirs. Later Reyes et al. applied this model to the production data from six Kern County oil fields and found that physical parameters derived from the decline curve analysis could be used to describe regional and reservoir properties.

The main purpose of this study was to conduct a comparison between the exponential model, the Li-Horne model, and the harmonic model. To do so, both experimental data from core samples and production data from reservoirs with different values of permeability were analyzed using the three models. Values of maximum recovery and recovery at an assumed economic limit were inferred using the three models and the results were compared. The model predictions were also compared to the true values in the cases of core samples.

Mathematical Background
The empirical Arps\(^1\) decline equation represents the relationship between production rate and time for oil wells during pseudosteady-state period and is shown as follows:

\[
q(t) = \frac{q_i}{(1 + bD_i t)^{1/b}}
\]  

(1)
where \( q(t) \) is the oil production rate at production time \( t \) and \( q_i \) is the initial oil production rate. \( b \) and \( D_i \) are two constants.

Eq. 1 can be reduced in two special cases: \( b=0 \) and \( b=1 \). \( b=0 \) represents an exponential decline and \( b=1 \) represents a harmonic decline in oil production. For \( 0< b<1 \), Eq. 1 is defined as the hyperbolic model. The two special cases of \( b=0 \) and \( b=1 \) will be discussed as follows.

The Exponential Model
For \( b=0 \), an exponential decline in oil production can be obtained and expressed as follows:

\[
q(t) = q_i e^{D_i t}
\]  
(2)

The exponential decline model can also be expressed in terms of cumulative production:

\[
N_p = \frac{1}{D_i} (q_i - q)
\]  
(3)

where \( N_p \) is the cumulative oil production. In the case of exponential decline, one should get a linear trend by plotting the cumulative oil production versus the production rate. According to Eq. 3, the maximum production can be obtained by setting \( q=0 \). The production at an economic limit of production (\( q_{\min} \)) can also be estimated from Eq. 3.

The Harmonic Model
In the Arps equation (Eq. 1), \( b=1 \) represents a harmonic decline in oil production, which can be expressed as follows:

\[
q(t) = \frac{q_i}{1 + D_i t}
\]  
(4)

In terms of cumulative production, the harmonic decline can be expressed as:

\[
N_p = -\frac{q_i}{D_i} \ln \left( \frac{q}{q_i} \right)
\]  
(5)

A linear trend between the cumulative oil production versus the logarithm of production rate will be expected according to Eq. 5. One can not infer the maximum production by setting \( q=0 \) from Eq. 5. However one may obtain the production at an economic limit of production (\( q_{\min} \)).

The Li-Horne Model
The Li and Horne production decline curve analysis model\(^{5}\) is expressed as follows:

\[
q(t) = a_0 \frac{1}{R(t)} - b_0
\]  
(6)

where \( R(t) \) is the recovery at time \( t \), in the units of pore volume \( (R=N_p/V_p, V_p \) is the pore volume). \( a_0 \) and \( b_0 \) are two constants associated with capillary and gravity forces respectively. The details on deriving Eq. 6 and calculating \( a_0 \) and \( b_0 \) were described by Li and Horne\(^{3,4}\).

The two constants \( a_0 \) and \( b_0 \) are expressed as follows\(^{7}\):

\[
a_0 = \frac{AM^* (S_{wf} - S_{wi})}{L}
\]  
(7)

\[
b_0 = AM^* \Delta \rho g
\]  
(8)

where \( A \) and \( L \) are the area and the characteristic length (or height) of the reservoir or the core, \( S_{wi} \) is the initial water saturation and \( S_{wf} \) is the water saturation behind the water front, \( \Delta \rho \) is the density difference between water (the wetting) and oil (the nonwetting) phases, \( g \) is the gravity constant, \( P_c^* \) is the capillary pressure at \( S_{wf} \) and \( M^* \) is the global mobility in which relative permeability data of oil and water are included.

In summary, all three models, the exponential model (Eq. 3), the harmonic model (Eq. 5), and the Li-Horne model (Eq. 6), represent the relationship between production rate and the cumulative production in different forms. If the decline trend is exponential, one should get a linear trend by plotting \( q \) versus cumulative production. If the decline trend is harmonic, we expect a linear trend by plotting the logarithm of \( q \) versus cumulative production. If the decline trend follows the Li-Horne mechanistic model (Eq. 6), one can obtain a linear trend by plotting \( q \) versus the reciprocal of cumulative production (or the recovery \( R \)). Therefore we can compare the three models by calculating the production at an economic production limit of \( q_{\min} \).

The maximum recovery or the recoverable reserve is defined as follows:

\[
R_{\max} = \frac{1-S_{wi}-S_{or}}{1-S_{wi}}
\]  
(9)

where \( S_{wi} \) is the initial water saturation and \( S_{or} \) is the residual oil saturation. Note that \( R_{\max} \) can be inferred from the exponential model and the Li-Horne model but not from the harmonic model.

Results
Both experimental data from flooding core samples and production data from reservoirs were used to verify and compare the three models. For all the comparisons, the production rate and recovery data applied were the same, including the number of data points. The advantage in using experimental recovery data from the core flooding experiments is that the reserves or the maximum production are known exactly. Therefore the accuracy of the production prediction models can be evaluated and compared.
High Permeability Sandstone

Fig. 1 shows the analysis results using the three models for experimental data of production from Berea sandstone (Li and Horne9). The Berea sandstone sample had a permeability of around 1200 md and a porosity of about 24.5%; its length and diameter were 43.5 cm and 5.06 cm. The Berea sandstone sample was fired at a high temperature to stabilize the clay.

The relationship between the production rate and the recovery was plotted in Fig. 1a to perform the exponential production decline analysis. There is a linear trend at the later period. The maximum recovery inferred using the exponential model was 0.555 OOIP (original oil in place). The recovery at an assumed economic limit of 0.001 g/minute calculated using the exponential model (Eq. 3) was also about 0.555 OOIP in this case. Note that the actual maximum recovery or the recoverable reserve calculated using Eq. 9 was 0.696 OOIP.

The difference between the prediction by the exponential model and the true value is significant. These data are also listed in Table 1 in which $R_{eco}$ represents the recovery at the assumed economic limit.

Fig. 1b shows a good linear trend of the relationship between the production rate and the reciprocal of recovery for most of the data points, as predicted by the Li-Horne model. The maximum recovery inferred using the Li-Horne model was 0.668 OOIP. The recovery at an assumed economic limit of 0.001 g/minute calculated using Eq. 6 was about 0.666 OOIP. The prediction by the Li-Horne model is close to the true value (0.696 OOIP) of the maximum recovery.

The relationship between the logarithm of production rate and recovery is shown in Fig. 1c, and was used to conduct the harmonic production decline analysis. The trend is also linear at the later period. As stated previously, the maximum recovery cannot be inferred from the harmonic model. The recovery at an economic limit of 0.001 g/minute calculated using Eq. 5 was 1.557 OOIP, which is far from the true value (0.696 OOIP) and is also physically impossible because the recovery cannot be greater than 1.0.

Note that the entire set of data points show a linear trend when the Li-Horne model is used but only the data points at the later period of production show a linear trend when the exponential and the harmonic models are applied.

Low Permeability Chalk

Figs. 2a, b, and c show the comparison of the three models in a low permeability chalk sample using the experimental recovery data of Li and Horne9. The permeability of the chalk sample was around 5 md and the porosity was 36.2%; its length and diameter were 7.5 cm and 2.54 cm, respectively.

As in the Berea cores, the Li-Horne model shows a better linear trend than the exponential and the harmonic models. The maximum recovery and the recovery at an economic limit were calculated and the results are listed in Table 1. The Li-Horne model gave the best estimation of the maximum recovery and the recovery at an economic limit, in comparison to the measured value. Note that the maximum recovery listed for the exponential model was inferred using only the data points that show a linear trend (see Fig. 2a). The magnitude would be much smaller if the entire data set were used as was done in Fig. 2b in which the Li-Horne model is applied.

Figs. 1 and 2 show the comparison of the three models using experimental recovery data obtained from flooding core samples. In order to further verify and compare the three models, oil production data from complex reservoirs developed by water flooding were also used and the results are shown in Fig. 3 through Fig. 5.

Naturally-Fractured Low Permeability Reservoirs

The oil production data by water flooding from the E.T. O’Daniel lease in Spraberry (Baker et al.8) were analyzed using the three models and the results are shown in Figs. 3a, b, and c. The Spraberry oilfield is a naturally-fractured low permeability reservoir with a high density of fractures. Water breakthrough occurred at production wells shortly after water injection began because of the high-density fractures. The oil recovery by water flooding in the oilfield is believed to be dominated by countercurrent water imbibition because of the early water breakthrough and the high-density fractures9.

One can see from Figs. 3a, b, and c that all of the three models show a linear trend at the later period of oil production. However the values of oil recovery predicted by the three models are different (see Table 1). For the oil recovery at an economic limit of 0.001 OOIP/year, the magnitudes calculated using the exponential, the Li-Horne, and the harmonic models was 0.358, 0.455, and 0.482 OOIP respectively. The prediction of recovery by the Li-Horne model is in between the exponential and the harmonic models, similar to the observation in the core samples.

Complex Reservoirs

Another example chosen was the oil production data in an offshore waterdrive field reported by Dake11 (p. 417). This oil field is not a naturally-fractured reservoir but has a large permeability contrast between layers. Water breakthrough happened at the early injection period because of the deltaic depositional environment and the large permeability contrast. The high permeability layers in between low permeability layers may function as fractures. Therefore the oil production data from this reservoir may also be analyzed using the Li-Horne model. The results obtained using the three models are shown in Figs. 4a, b, and c. Features similar to Fig. 3 are observed. Data points plotted using all three models show a linear trend. However the inferred values of the oil recovery at an economic limit of 0.001 OOIP/year were different and were 0.348, 0.535, and 0.683 OOIP respectively (see Table 1). The predicted maximum oil recovery was about 0.351 OOIP when the exponential model was used and was 0.552 OOIP when the Li-Horne model is used. Note that the maximum oil recovery estimated by Dake11 (p. 417) using a different technique was about 0.554 OOIP, which is very close to the result predicted by the Li-Horne model.

Fault Reservoirs

The last example chosen is the oil production data after water breakthrough in another North Sea oil field developed by water flooding, also reported by Dake11 (p. 443). This is an isolated fault block of an extremely complex oil field. It is of the delta top depositional environment with a large permeability contrast. Dake11 (p. 443) reported that numerical simulation modeling failed to produce a history match to the
field performance. Figs. 5a, b, and c demonstrate the results analyzed using the three models. Both the exponential and the Li-Horne models match the production data satisfactorily and predict similar maximum oil recovery, 0.236 and 0.239 OOIP respectively. It is difficult to find a linear trend using the harmonic model (see Fig. 5c).

We have also conducted comparisons using other production data, for example, the oil production data in East Texas oil field reported by Dake\textsuperscript{11} (p. 450). A similar phenomenon was observed: all of the three models can match the production data at the later period of production. However the values of oil recovery predicted by the three models are different. The results are listed in Table 1 although the graphs are not presented here.

**Comparison of the Three Models**

Fig. 6a shows the comparison between the exponential and the Li-Horne models in terms of maximum recovery. The values of maximum recovery predicted by the Li-Horne model are greater than those predicted by the exponential model. Note that the exponential decline curve tends to underestimate reserves and production rates. The number of reservoir in Fig. 6a is defined in Table 1.

Fig. 6b shows the comparison among the three models in terms of the recovery at an economic limit. One can see that the magnitude predicted using the Li-Horne model is in between the exponential and the harmonic models, greater than the exponential model but less than the harmonic model. As stated previously, the harmonic model has a tendency to overpredict the production (Agbi and Ng\textsuperscript{3}). The values of the recovery at an assumed economic limit inferred using the harmonic model are greater than one pore volume in several cases, which is physically impossible.

The comparison presented in Figs. 6a and b demonstrates that the Li-Horne model may have a better accuracy than the exponential and the harmonic models for the cases studied.

**Conclusions**

Based on the present study, the following conclusions may be drawn in the cases studied:

1. The maximum recovery determined in core samples using the Li-Horne model is close to the true value, whereas the results estimated using the exponential model are significantly different from the true.
2. The harmonic model overestimates the recovery in many cases. The recovery (in the units of OOIP at an economic limit) inferred using the harmonic model was greater than 1.0 in some cases, which is physically impossible.
3. The exponential model underestimates the recovery or the reserve, which is verified using the experimental data in both low and high permeability core samples.
4. The recovery at an assumed economic limit predicted using the Li-Horne model was greater than the exponential model but was less than the harmonic model in all of the cases studied.
5. The comparison of model predictions using the experimental recovery data in the core samples demonstrated that the Li-Horne model had the best estimation of the recoverable recovery, compared to the exponential and the harmonic models in the cases studied.

**Acknowledgements**

This research was conducted with financial support from the US Department of Energy under grant DE-FG07-02ID14418, the contribution of which is gratefully acknowledged.

**Nomenclature**

- \( a_0 \) = coefficient associated with capillary forces, m/t
- \( A \) = cross-section area of the core or reservoir, L\(^2\)
- \( b_0 \) = coefficient associated with gravity, m/t
- \( D \) = constant
- \( g \) = gravity constant, L/t\(^2\)
- \( k \) = absolute permeability, L\(^2\)
- \( k_r^* \) = relative permeability of oil phase at a specific water saturation
- \( k_w^* \) = relative permeability of water phase at a specific water saturation
- \( L \) = core length, L
- \( M_e^* \) = global mobility of the two phases at a specific water saturation, mL/t
- \( M_o^* \) = mobility of oil phase at a specific water saturation, mL/t
- \( M_w^* \) = mobility of water phase at a specific water saturation, mL/t
- \( N_p \) = cumulative production rate, L\(^3\)/t
- \( P_c^* \) = capillary pressure at a specific water saturation, m/Lt\(^2\)
- \( q \) = oil production rate, L\(^3\)/t
- \( q_i \) = initial oil production rate, L\(^3\)/t
- \( R \) = oil recovery in the units of pore volume
- \( R_{max} \) = maximum oil recovery in the units of pore volume
- \( S_{ref} \) = water saturation behind imbibition front
- \( S_{wi} \) = initial water saturation
- \( t \) = production time, t
- \( \mu_o \) = viscosity of oil phase, mL/t
- \( \mu_w \) = viscosity of water, mL/t
- \( \rho_o \) = density of oil phase, mL\(^3\)
- \( \rho_w \) = density of water phase, mL\(^3\)
- \( \Delta \rho \) = density difference between water and oil phases, mL\(^3\)

**References**

Western Regional Meeting, Long Beach, California, 19–24 May 2003.


Table 1: Values of recovery predicted by the three models

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>Exponential</th>
<th>Li-Horne</th>
<th>Harmonic</th>
<th>Measured</th>
<th>Note</th>
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<tr>
<td></td>
<td></td>
<td>$R_{\text{max}}$</td>
<td>$R_{\text{eco}}$</td>
<td>$R_{\text{max}}$</td>
<td>$R_{\text{eco}}$</td>
<td>$R_{\text{eco}}$</td>
</tr>
<tr>
<td>1</td>
<td>Fired Berea</td>
<td>0.555</td>
<td>0.555</td>
<td>0.668</td>
<td>0.666</td>
<td>1.557</td>
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<td>2</td>
<td>Chalk</td>
<td>0.784</td>
<td>0.760</td>
<td>0.827</td>
<td>0.769</td>
<td>1.062</td>
</tr>
<tr>
<td>3</td>
<td>Offshore field</td>
<td>0.351</td>
<td>0.348</td>
<td>0.552</td>
<td>0.535</td>
<td>0.683</td>
</tr>
<tr>
<td>4</td>
<td>North Sea fault</td>
<td>0.236</td>
<td>0.235</td>
<td>0.239</td>
<td>0.238</td>
<td>0.273</td>
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<tr>
<td>5</td>
<td>East Texas field</td>
<td>0.606</td>
<td>0.592</td>
<td>0.651</td>
<td>0.626</td>
<td>0.748</td>
</tr>
<tr>
<td>6</td>
<td>East Texas field</td>
<td>0.869</td>
<td>0.843</td>
<td>0.966</td>
<td>0.914</td>
<td>1.050</td>
</tr>
<tr>
<td>7</td>
<td>Spraberry field</td>
<td>0.377</td>
<td>0.358</td>
<td>0.536</td>
<td>0.455</td>
<td>0.482</td>
</tr>
</tbody>
</table>

*Recovery is in the units of OOIP.

![Fig. 1a: Analysis using the exponential model for experimental data from Berea sandstone.](image1)

![Fig. 1b: Analysis using the Li-Horne model for experimental data from Berea sandstone.](image2)
Fig. 1c: Analysis using the harmonic model for experimental data from Berea sandstone.

Fig. 2a: Analysis using the exponential model for experimental data from chalk.

Fig. 2b: Analysis using the Li-Horne model for experimental data from chalk.

Fig. 3a: Analysis using the exponential model for production data from Spraberry oil field.

Fig. 3b: Analysis using the Li-Horne model for production data from Spraberry oil field.
0.00 0.02 0.04 0.06 0.08 0.10
Recovery, OOIP

0.00 0.01 0.02 0.03 0.04
Production Rate, OOP/Year

Fig. 3c: Analysis using the harmonic model for production data from Spraberry oil field.

0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40
Recovery, OOIP

0.00 0.02 0.04 0.06 0.08 0.10
Production Rate, OOP/Year

Fig. 4a: Analysis using the exponential model for an offshore oil field with a high permeability contrast.

0.00 0.02 0.04 0.06 0.08 0.10
Recovery, OOIP

0.00 0.02 0.04 0.06 0.08 0.10
Production Rate, OOP/Year

Fig. 4b: Analysis using the Li-Horne model for production from an offshore oil field with a high permeability contrast.

0.00 0.10 0.20 0.30 0.40
Recovery, OOIP

0.00 0.01 0.02 0.03 0.04
Production Rate, OOP/Year

Fig. 4c: Analysis using the harmonic model for production from an offshore oil field with a high permeability contrast.

0.00 0.02 0.04 0.06 0.08 0.10
Recovery, OOIP

0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40
Recovery, OOIP

0.00 0.02 0.04 0.06 0.08 0.10
Production Rate, OOP/Year

Fig. 5a: Analysis using the exponential model for production data from a North Sea fault oil field.

0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40
Recovery, OOIP

0.00 0.02 0.04 0.06 0.08 0.10
Production Rate, OOP/Year

Fig. 5b: Analysis using the Li-Horne model for production data from a North Sea fault oil field.
Fig. 5c: Analysis using the harmonic model for production data from a North Sea fault oil field.

Fig. 6a: Comparison between the exponential model and the Li-Horne model.

Fig. 6b: Comparison among the exponential, the harmonic, and the Li-Horne models.